long as this initial horizontal plane remains a vertical plane, the sphere will slide without rolling.

**25. Proposed by Professor GEORGE LILLEY, LL. D., Ex-President of Washington State Agricultural College and School of Science, Portland, Oregon.**

It is known that if the velocity of a certain freight train is 30 miles an hour it can be brought to a stand still in a distance of 500 feet by setting the brakes. It was stopped in 1200 feet by setting the brakes. Find its velocity, the forces exerted by the brakes being the same in each case.

**Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Physics, Inter State College, Texarkana, Texas, and the PROPOSER.**

$\frac{1}{2}Mv^2=Rs$, where $M$=mass, $v$=velocity, $R$=resistance of brakes, $s$=distance train runs after setting brakes.

30 miles per hour=44 feet per second.

$\therefore \frac{1}{2}M(44)^2=500R \ldots (1)$. $\frac{1}{2}Mv^2=1200R \ldots (2)$.

$$(2)\div(1),\quad \left(\frac{v}{44}\right)^2=\frac{12}{5}. \quad \therefore 5v^2=12(44)^2.$$

$\therefore v=88\sqrt{\tfrac{3}{5}}$ feet per second $=60\sqrt{\tfrac{3}{5}}=46.4758$ miles per hour.

Also solved by *F. P. MATZ* and *E. W. MORRELL*.

## PROBLEMS.

**31. Proposed by O. W. ANTHONY, Mexico, Mo.**

A perfectly elastic but perfectly rough sphere of mass $M$ and radius $R$, rotating in a vertical plane with an angular velocity of $\omega$, is let fall from a height, $a$, upon a perfectly elastic but perfectly rough horizontal plane. Determine the motion of the body after striking the plane. What will be its ultimate motion?

**32. Proposed by OTTO CLAYTON, A. B., Fowler, Indiana.**

The wheel of a wind pump has 60 fans, each turned at an angle 45° to the direction of the axis, and each having 150 square inches exposed to the wind. If the wind blows with velocity $V$ and the wheel rotates with velocity $\omega$ what is the component of force or pressure along the axis if it is turned at an angle $\alpha$ to the direction of the wind assuming the resistance the wheel meets in turning to be $R$?

# DIOPHANTINE ANALYSIS.

**Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.**

## SOLUTIONS OF PROBLEMS.

**25. Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.**

Find, if possible, integral values of each of the seven linear measurments of a

rectangular parallolopiped; i. e. length, breadth, height, the diagonals of each of the three different rectangular sides, and the diagonal from an upper corner to the opposite lower corner; or, find integral values, if possible, of $a$, $b$, $c$, $d$, $e$, $f$, and $g$, as shown in the equations,— $a^2+b^2=c^2$, $a^2+d^2=e^2$, $a^2+f^2=g^2$, $b^2+d^2=f^2$, $b^2+e^2=g^2$, $c^2+d^2=g^2$, $c^2+e^2=f^2$. If not possible, how many of them can have integral values? and which?

**Solution by G. B. M ZERR, A. M., Ph. D., Vice-President and Professor of Mathematics and Sciences, Inter State College, Texarkana, Texas.**

Let the length, breadth, and height be, $a=8mn(m^4-n^4)$, $b=2mn\{10m^2n^2-3(m^4+n^4)\}$, $c=(m^2-n^2)(m^4+n^4-14m^2n^2)$.

The method of obtaining the above values has been published in several journals and need not be repeated here. From the above we easily get $a^2+b^2=\{2mn(5m^4-6m^2n^2+5n^4)\}^2$, $a^2+c^2=(m^6+17m^4n^2-17m^2n^4-n^6)^2$, $b^2+c^2=(m^6+3m^4n^2+3m^2n^4+n^6)^2=(m^2+n^2)^6$, and $a^2+b^2+c^2=64m^2n^2(m^4-n^4)^2+(m^2+n^2)^6$. This last is a square when $64m^2n^2(m^2-n^2)^2+(m^2+n^2)^4=\square$. Let $m=pn$. Then must $64p^2(p^2-1)^2+(p^2+1)^4=p^8+68p^6-122p^4+68p^2+1=\square$.

I have not yet succeeded in making this last a square. The edges and diagonals of sides, are integral, satisfying six of the relations.

Let $m=2$, $n=1$; then $a=240$, $b=44$, $c=117$, $\sqrt{a^2+b^2}=244$, $\sqrt{a^2+c^2}=267$, $\sqrt{b^2+c^2}=125$, $\sqrt{a^2+b^2+c^2}=5\sqrt{2929}$.

**26. Proposed by F. P. MATZ, D. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.**

Find (1) a *square fraction* the arithmetical difference of whose terms is a *cube*; and (2) find a *cubic fraction* the arithmetical sum of whose terms is a *square*.

**I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.**

(1) Let $\frac{y^2}{x^2}$ equal the *square fraction*. Then $x^2-y^2=a$ *cube*$=a^3$. Then $(x+y)(x-y)=a^3$. Put $x+y=a^2$, and $x-y=a$.

$$\text{Then } x=\frac{a^2+a}{2}=\frac{a(a+1)}{2}; \text{ and } y=\frac{a^2-a}{2}=\frac{a(a-1)}{2}.$$

Whatever integral values be assigned to $a$, $x$ and $y$ will always be integral; for whether $a$ is even or odd, $a(a+1)$ and $a(a-1)$ are *even*.

$$\text{Since } x-y=a,\ \frac{a(a-1)}{2}+a=\frac{a(a+1)}{2}.$$

$\therefore$ the denominator of the fraction $\frac{y}{x}$ is always $a$ more than the numerator. Also, as $\frac{a(a-1)}{2}$ is the sum of the series $(1+2+3+\ldots.+a-1)$, $\frac{y}{x}=(1+2+3+\ldots.+a-1)\div[(1+2+3+\ldots.+a-1)+a]$, or $(1+2+3+\ldots.+a-1)\div(1+2+3+\ldots.+a)$. Putting $a$ equal, consecutively, to the successive